Chapter 2: Transformers

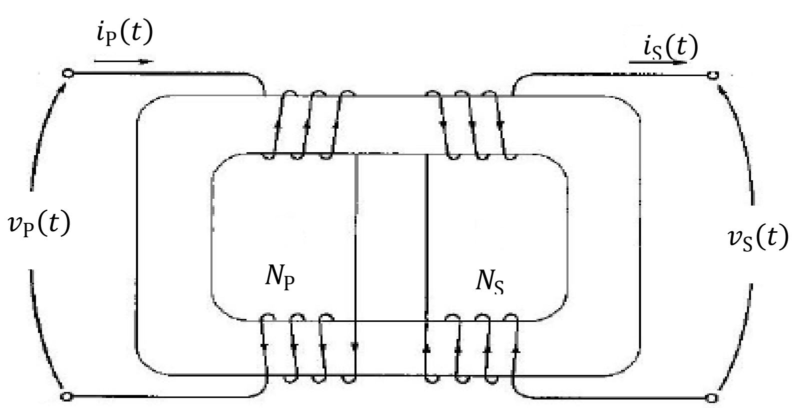
Function: **To convert ac power** at one voltage level to ac power of the same frequency at another voltage level.

2.1 Types and Construction of Power Transformers

Types	Descriptions
Unit Transformer	• Connected to the output of a generator • Used to step up generator output voltage to transmission levels (275 kV)
Substation Transformer	• Connected at the other end of the transmission lines • Used to step down voltage from transmission levels to distribution levels (33 kV)
Distribution Transformer	• Used to step down the voltage at distribution levels to final voltage (220V, 110 V)

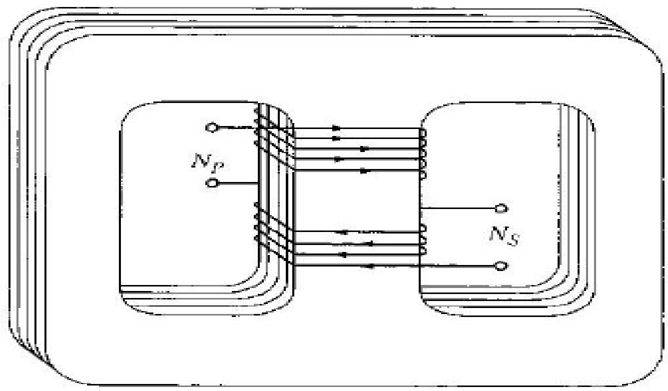
A transformer consists of two or more windings wrapped around a common ferromagnetic core.

1) **Core-type transformer** is made up of a simple rectangular laminated piece of steel. The windings are wrapped around both sides (primary and secondary) of the core.



Note that by convention,
Primary side: connected to the input/source
Secondary side: connected to the output/load

2) **Shell-type transformer** is made up of a three-legged laminated piece of steel. The windings are wrapped around the center leg.



In either case, both transformer types are constructed of thin laminations electrically isolated from each other in order to minimize eddy currents.

In a **physical transformer**, the primary and secondary windings are wrapped one on top of the other with the low-voltage winding innermost to serve two purposes:

1) To simplify the problem of insulating the high- voltage winding from the core

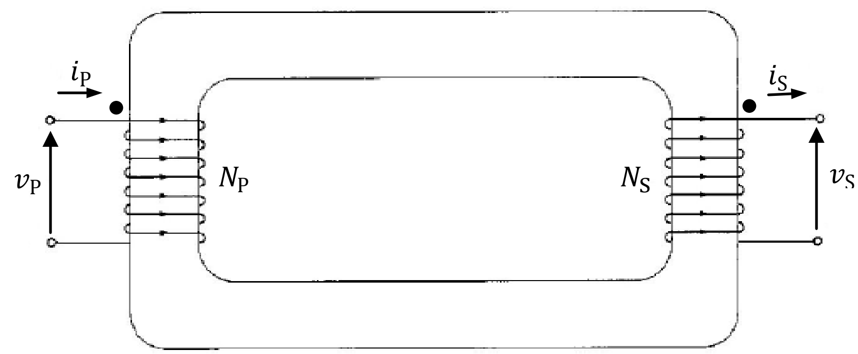
2) To reduce leakage flux

Having said so, the shell-type transformer is more advantageous than the core-type transformer.

2.2 The Ideal Transformer

Conditions for an ideal transformer:

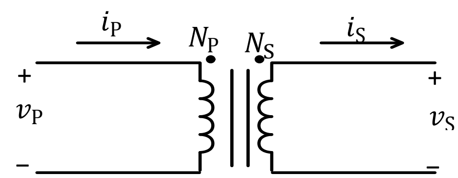
- Lossless or non-dissipative
- No energy storage

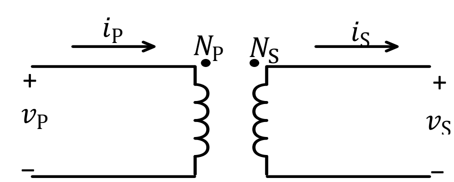


Assumptions made for an ideal transformer:

1) No loss in the winding resisistance
2) No leakage flux
3) Core is infinitely permeable and has no loss

<u>Schematics symbol of a transformer</u>





The **dot convention** in the schematic diagram of the transformer has the following relationship:

1) If $\boldsymbol{v_p}$**= +ve** at the dotted end, then $\boldsymbol{v_s}$ **= +ve** at the dotted end. **Voltage polarities** are the **same** wrt the dots on each side of the core.

2) If $\boldsymbol{i_p}$ flows **into** the dotted end, then $\boldsymbol{i_s}$ will flow **out** of the dotted end.

Refer to the sketch of the ideal transformer

When an ac voltage v_p is applied at the primary windings, according to Faraday's Law, a core flux must be established such that the counter emf $e_p = v_p$ at steady state. Therefore at steady state,

$$v_P = e_P = N_P \frac{d\emptyset_P}{dt}$$

The core flux also links the secondary winding and produces and emf e_S, and an equal secondary terminal voltage v_S as shown,

$$v_S = e_S = N_S \frac{d\emptyset_S}{dt}.$$

From the ratio below,

$$\frac{v_P}{v_S} = \frac{e_P}{e_S} = \frac{N_P}{N_S} = a.$$

Where *a* is defined as the *turns ratio* of the transformer.

If a load is connected on the secondary side, a current i_2 and an mmf $N_2 i_2$ are present. Since the core permeability is infinite, the **net exciting mmf** acting on the core will not change and remains negligible as shown as,

The relationship of current and turns ratio of the transformer is,

Note that the **turns ratio** of the ideal transformer **only affects** the **magnitudes** of current and voltage phasors. **Phase angles** are **not affected**.

In phasor form the voltage and current relationships with the turns ratio are

$$\frac{\bar{V}_{\mathrm{P}}}{\bar{V}_{\mathrm{S}}} = \frac{N_{\mathrm{P}}}{N_{\mathrm{S}}} = a$$

and

$$\frac{\bar{I}_{\mathrm{P}}}{\bar{I}_{\mathrm{S}}} = \frac{N_{\mathrm{S}}}{N_{\mathrm{P}}} = \frac{1}{a}$$

2.2.1 Power in an Ideal Transformer

Input power at the primary side is given as,

$$P_{\mathrm{in}}=V_{\mathrm{P}} I_P cos\ \theta_{\mathrm{P}}\ ,$$

Output power at the secondary side is given as,

$$P_{\mathrm{out}}=V_{\mathrm{S}} I_{\mathrm{S}} cos\ \theta_{\mathrm{S}}\ ,$$

where θ_{P} and θ_{S} are the phase angles between the voltage and current in the primary and secondary sides, respectively.

The **phase angles are unaffected** by the transformer turns ratio. Therefore,

$$\theta_{\mathrm{P}} = \theta_{\mathrm{S}} = \theta$$

Since $\boldsymbol{V}_{\mathbf{S}} = \boldsymbol{V}_{\mathbf{P}}/\boldsymbol{a}$ and $\boldsymbol{I}_{\mathbf{S}} = \boldsymbol{a}\boldsymbol{I}_{\mathbf{P}}$, the output power in the secondary side can be replaced and represented as,

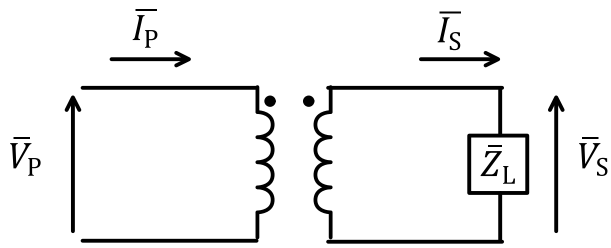
The reactive power Q and apparent power S can be determined in the same manner to be

$$Q_{\mathrm{in}} =\ V_{\mathrm{S}} I_{\mathrm{S}} \sin\theta_{\mathrm{P}} = V_{\mathrm{S}} I_{\mathrm{S}} \sin\theta_{\mathrm{S}} = Q_{\mathrm{out}}$$

and

$$S_{\text{in}} = V_{\text{P}} I_{\text{P}} = V_{\text{S}} I_{\text{S}} = S_{\text{out}}$$

2.2.2 Impedance Transformation Through a Transformer



The load impedance as seen at the secondary side is given as

$$\bar{Z}_{\text{L}} = \frac{\bar{V}_{\text{S}}}{\overline{I_S}}$$

The apparent impedance as seen at the primary side is,

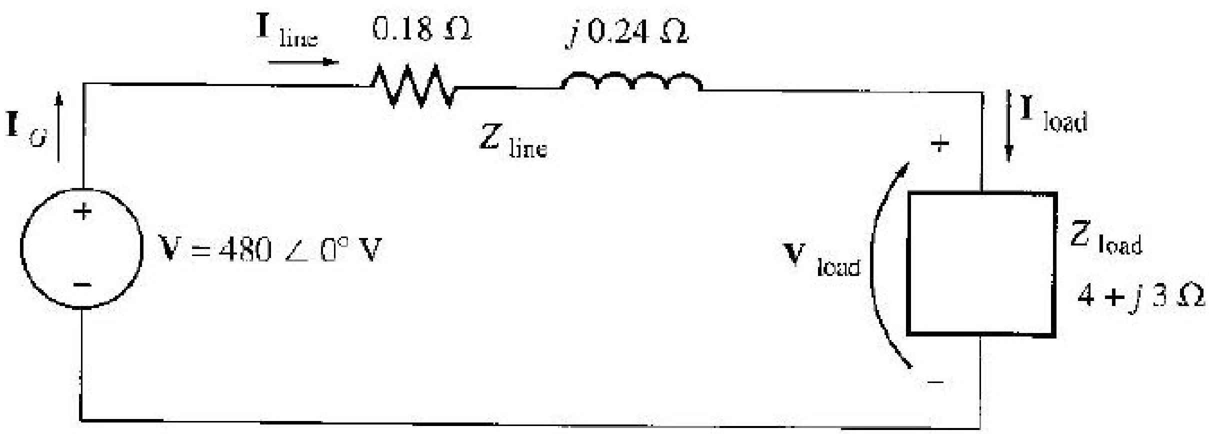
$$\bar{Z}'_{\text{L}} = \frac{\bar{V}_{\text{P}}}{\overline{I_{\text{P}}}} = \qquad =$$

The primary-side-referred transformer equivalent circuit is,

2.2.3 Analysis of circuit containing ideal transformer

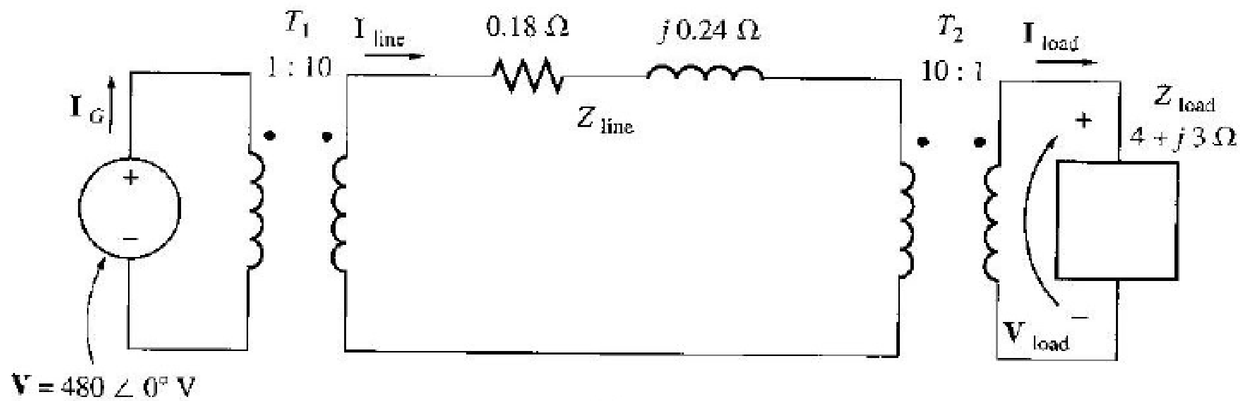
Example 2.1

A generator rated at 480V, 60 Hz is connected a transmission line with an impedance of 0.18+j0.24Ω. At the end of the transmission line there is a load of 4+j3Ω.



a) Calculate the voltage at the load and the transmission losses.

b) Suppose a 1:10 step-up transformer is placed at the generator end of the transmission line and a 10:1 step-down transformer is placed at the load end of the line. Calculate the voltage at the load and the transmission losses.



To analyse this power system, we need to convert to a common voltage level.

1. Eliminate transformer T2 by referring load to the transmission line's voltage level:

2. Recalculate total equivalent impedance at the transmission side:

3. Eliminate transformer T1 by referring the total equivalent impedance to the generator side:

Now we can calculate generated current,

Having known this, we can work back to find the transmission line and load currents.
Through T1:

Through T2:

Therefore, the load voltage is:

Finally, the transmission line losses are:

2.3 Theory of Operation of Real Single-Phase Transformers

A real transformer has:

1. Leakage flux
2. Magnetizing current
3. Core losses (hysteresis and eddy current losses)

The basis of transformer operation can be derived from Faraday's Law (Chapter 1)

$$e_{\text{ind}} = \frac{d\lambda}{\mathrm{dt}}$$

Where λ is the flux linkage in the transformer coil as shown as,

$$\lambda = \sum_{i=1}^{N} \phi_i$$

The above relation is true provided on the assumption that the flux passing through each turn is constant. But in reality, the flux value at each turn may vary due to the position of the coil itself.

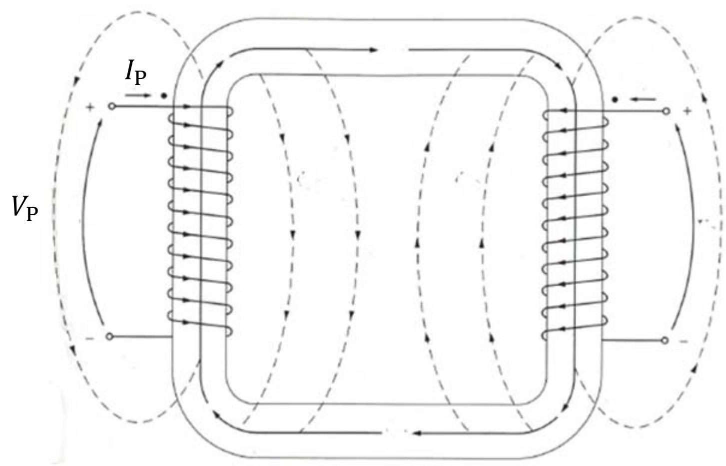
Therefore, the average flux per turn can be used as,

$$\phi_{\text{av}} = \frac{\lambda}{N}$$

and the Faraday's Law can be rewritten as

$$e_{\text{ind}} = N\frac{d\phi_{\text{av}}}{dt}$$

2.3.1 Transformer Leakage Flux



Not all the flux produced in the primary coil passes through the secondary coil. The portion of the flux that goes through one coil but not the other is called *leakage flux*.

ϕ_{LP} is defined as the primary leakage flux
ϕ_{LS} is defined as the secondary leakage flux

Flux linking both the primary and secondary coils and remains in the core, ϕ_M is defined as mutual flux.

The total **average primary flux** $\phi_{P(av)}$is the sum of primary leakage flux and the mutual flux, where

$$\phi_{P(av)} = \phi_{LP} + \phi_M.$$

Similarly, the total average secondary flux $\phi_{S(av)}$ is the sum of the secondary leakage flux and the mutual flux, where

$$\phi_{S(av)} = \phi_{LS} + \phi_M.$$

Therefore, Faraday's Law for the primary side is expressed as,

$$v_P = N_P \frac{d\phi_{P(av)}}{dt} = N_P \frac{d\phi_M}{dt} + N_P \frac{d\phi_{LP}}{dt}$$

$$v_P = e_P + e_{LP}$$

and, Faraday's Law for the secondary side is expressed as,

$$v_{\mathrm{S}} = N_{\mathrm{S}}\frac{d\phi_{\mathrm{S(av)}}}{dt} = N_{\mathrm{S}}\frac{d\phi_{\mathrm{M}}}{dt} + N_{\mathrm{S}}\frac{d\phi_{\mathrm{LS}}}{dt}$$

$$v_{\mathrm{S}} = e_{\mathrm{S}} + e_{\mathrm{LS}}$$

The mutual flux linkage is common to both the primary and secondary side, where

$$\frac{e_{\mathrm{P}}}{N_P} = \frac{d\phi_{\mathrm{M}}}{dt} = \frac{e_{\mathrm{S}}}{N_{\mathrm{S}}}$$

The ratio of induce primary voltage to the induced secondary voltage due to the mutual flux is

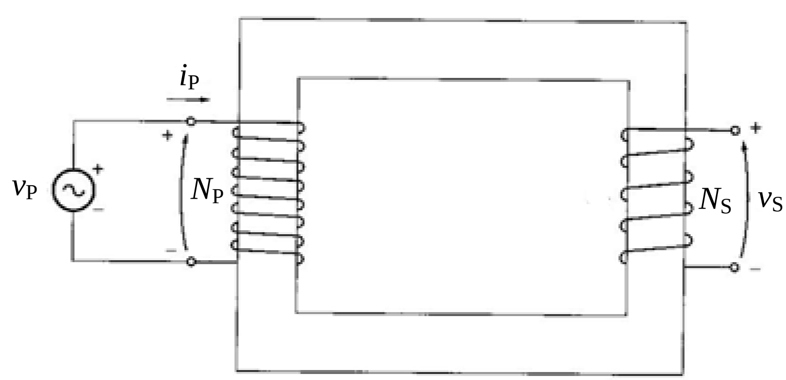
$$\frac{e_{\mathrm{P}}}{e_S} = \frac{N_{\mathrm{P}}}{N_S} = a$$

A good transformer design will be have negligible leakage flux., where $\phi_{\mathrm{M}} \gg \phi_{\mathrm{LP}}$ and $\phi_{\mathrm{M}} \gg \phi_{\mathrm{LS}}$. So, the total voltage ratio of the primary side to the secondary side is approximated as

$$\frac{v_{\mathrm{P}}}{v_{\mathrm{S}}} = \frac{N_{\mathrm{P}}}{N_S} = a$$

2.3.2 The Magnetization and Core-Loss Currents

When an ac **power** is **connected to primary side** of the transformer with the **secondary side open-circuited**, there is a current i_{P} that flows in the primary side and is known as the no-load current or the **transformer excitation current $\boldsymbol{i}_{\mathbf{ex}}$**.



The transformer excitation current consists of two current components. They are:

1. **Magnetization current** i_{M}, which is the current that is required to produce flux in the transformer core
2. **Core-loss current** $i_{\mathrm{h+e}}$, which is the current required to make up for hysteresis and eddy current losses.

$$\therefore i_{\mathrm{ex}} = i_{\mathrm{M}} + i_{\mathrm{h+e}}$$

Let us consider the magnetization current.

We know that in theory the relation between current and flux is proportional since,

$$F = \phi R = Ni$$

$$i = \frac{\phi R}{N}$$

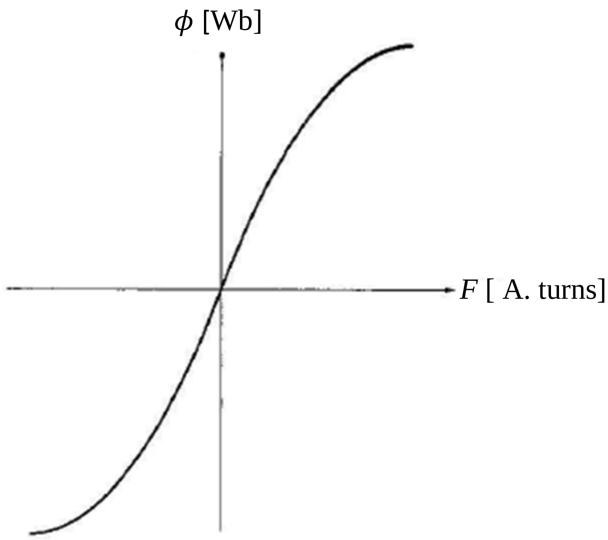
Therefore, in theory, if the flux produce in core is sinusoidal, therefore the current should also be a perfect sinusoidal. Unfortunately, this is **true only within the linear region** of the flux (ϕ)–magnetomotive force (F) relationship. However, it is not true for transformer flux that reaches a state of near saturation at the top of the flux cycle. Hence at this point, more current is required to produce a certain amount of flux.

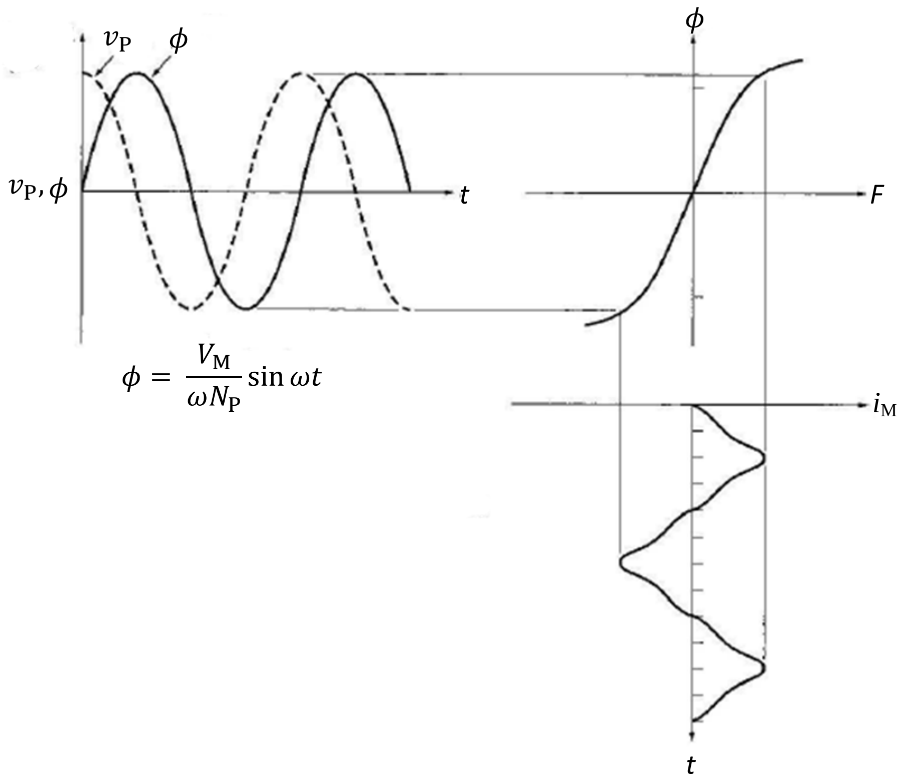
We know from before, that an average flux in the core is given as,

$$\bar{\phi} = \frac{1}{N_{\mathrm{P}}} \int v_{\mathrm{P}}\, dt$$

If the primary voltage is given as $v_{\mathrm{P}} = V_{\mathrm{M}} \cos \omega t$ V, then the resulting flux is,

The resulting flux is lagging by 90° to the primary voltage.





Based on the flux-time relationship, a sketch of the magnetization current in the windings can be carried out as shown above.

The magnetization current has the following characteristics:

1. It is **not sinusoidal**. It has higher frequency components due to magnetic saturation in the transformer core.

2. At saturation flux, a **large increase in current** is required **for a small increase in flux**.
3. Fundamental component of the magnetization **current lags the voltage** applied **by 90°**.
4. The higher-frequency components in the magnetization current can be larger than the fundamental. It can cause excessive heating in the transformer.

Let us consider the core-loss current

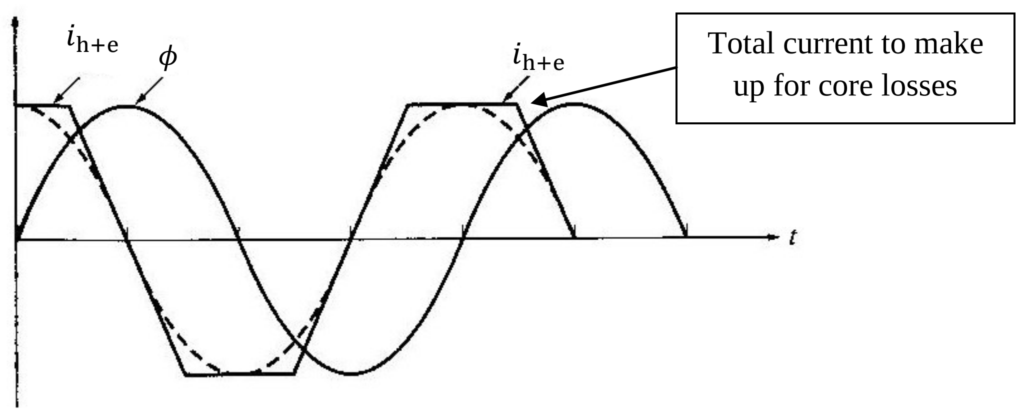
Core-loss current is required to compensate for:

1. Hyteresis and
2. Eddy-current losses.

Hysteresis loss is due to work done in the core by the magnetic field.

Eddy-current loss is the loss due to the eddy-current generated in the core.

Eddy current flow in directions such that it produces a magnetic field that opposes the applied magnetic field. The eddy current is proportional to the rate of change of flux.



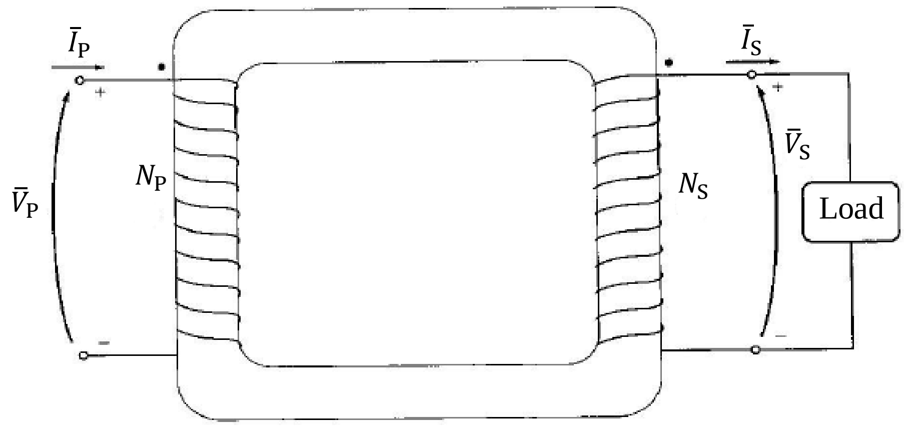
Therefore, the characteristics of the core-loss current are:

1. **Non-linear** because of the non-linear effects of hysteresis
2. The **fundamental component** of the core-loss current is **in phase with** the **voltage applied** to the core.

2.3.3 The Current Ratio on a Transformer and the Dot Convention

The dots help determine the polarity of the voltages and currents in the core without having to physically examine the windings.

A current flowing into the dotted end of a winding produces a 'positive' mmf, while a current flowing into the undotted end produces a 'negative' mmf .



Now, when we connect a load to the real transformer, the primary current will produce a positive mmf.

Due to Lenz's law, the secondary current will flow out of the dotted end to produce a negative mmf: (current flow will be in a direction as such to oppose the core flux direction).

Therefore, the net mmf in the transformer required to produce flux in the core is given by,

$$F_{\text{net}} = N_P i_P - N_S i_S = \phi R$$

where R is the reluctance of the transformer core. $R \approx 0$ provided the core is **unsaturated**, hence

$$F_{\text{net}} = N_P i_P - N_S i_S = 0$$

Therefore, the primary side mmf is approximately equal to the secondary side mmf whereby,

$$N_P i_P \approx N_S i_S.$$

The expression above can be rearranged so that

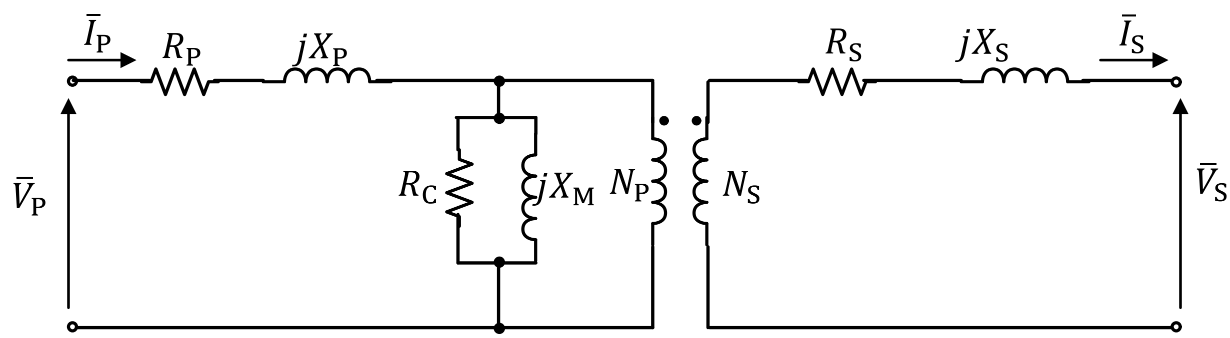
$$\frac{i_{\mathrm{P}}}{i_{\mathrm{S}}} \approx \frac{N_{\mathrm{S}}}{N_{\mathrm{P}}} = \frac{1}{a}$$

In order for the magnetomotive force to be nearly zero, current must flow into one dotted end and out of the other dotted end.

The first approach in transformer design can be made simple by assuming an ideal transformer. In an ideal transformer:

1. The core must have no hysteresis or eddy current
2. The net mmf F_{net} must be zero
3. There is no leakage flux i.e. all flux couples both windings
4. The transformer winding resistance must be zero.

2.4 The Equivalent Circuit of a Transformer



The transformer model above considers the following:

1. **Copper losses**–resistive heating losses in the primary and secondary windings of the transformer. Winding resistance represented by R_{P} and R_{S}.

2. **Eddy current losses**–resistive heating losses in the core of the transformer. The loss is proportional to square of the primary voltage.

3. **Hysteresis loss**–related to rearrangement of domains in core and is a complex, non-linear function of primary voltage.

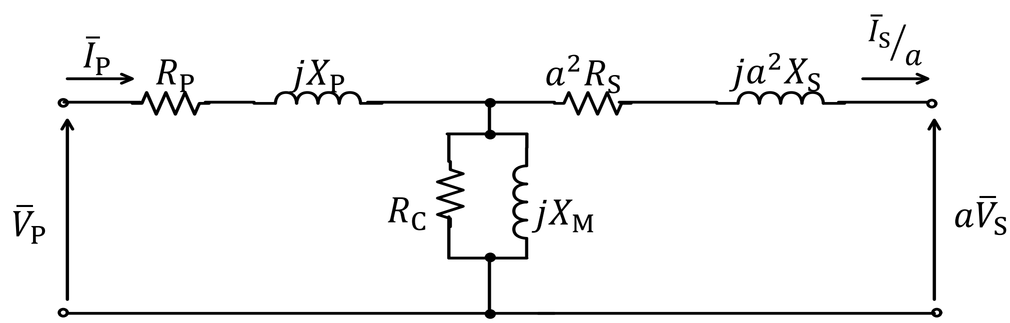
4. **Magnetization current**–proportional to the voltage applied (unsaturated region) but lagging the applied voltage by 90°. So it can be modeled as a reactance $X_\mathrm{M}$connected across the primary voltage source.

Note that: The core-loss current and the magnetization current are actually nonlinear. The inductance X_M and the resistance R_C are approximations of the real excitation effect to simplify analysis.

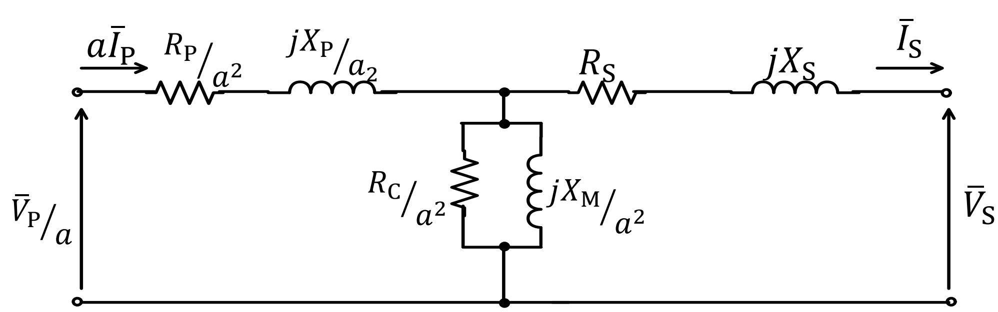
5. **Leakage flux**–fluxes that escape and do not link both windings are modelled as self-inductances connected in series with primary and secondary circuit, X_P and X_S

In order **to analyze practical transformer circuits**, it is necessary to c**onvert the equivalent circuit** with two voltage levels **to that of a single voltage level** by referring it to the primary or secondary side.

Primary referred transformer equivalent circuit



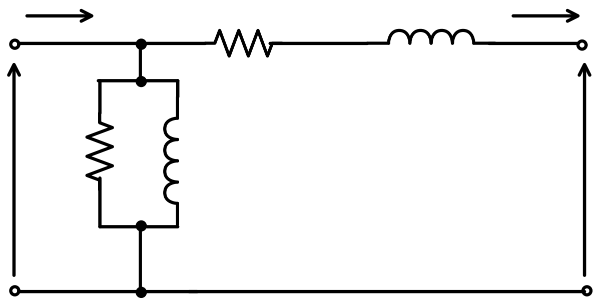
Secondary referred transformer equivalent circuit



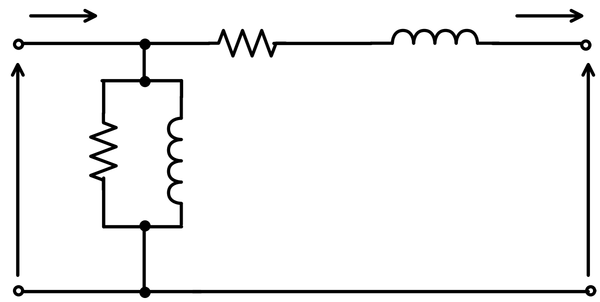
Further simplification of the transformer equivalent circuit can be carried out:

In practical situations, $R_\mathrm{C} \gg R_\mathrm{P}$ and $X_\mathrm{M} \gg X_\mathrm{P}$

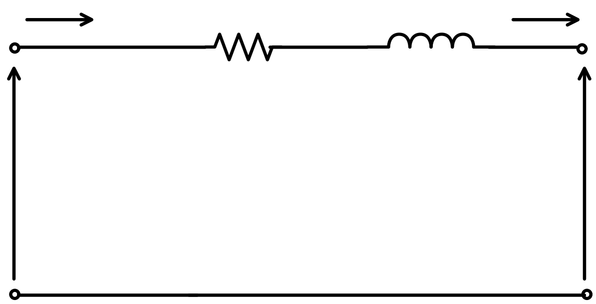
Therefore the voltage drop across R_{P} and X_{P} is negligible and the core excitation branch can be moved to the front as shown:



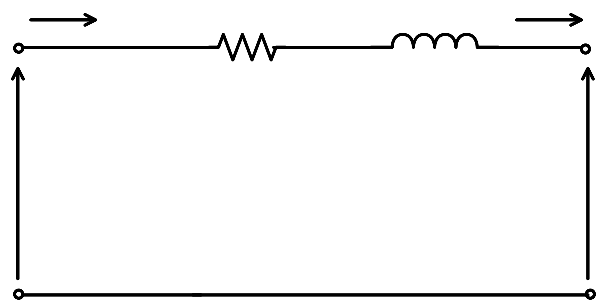
(a) Referred to the primary side



(b) Referred to the secondary side



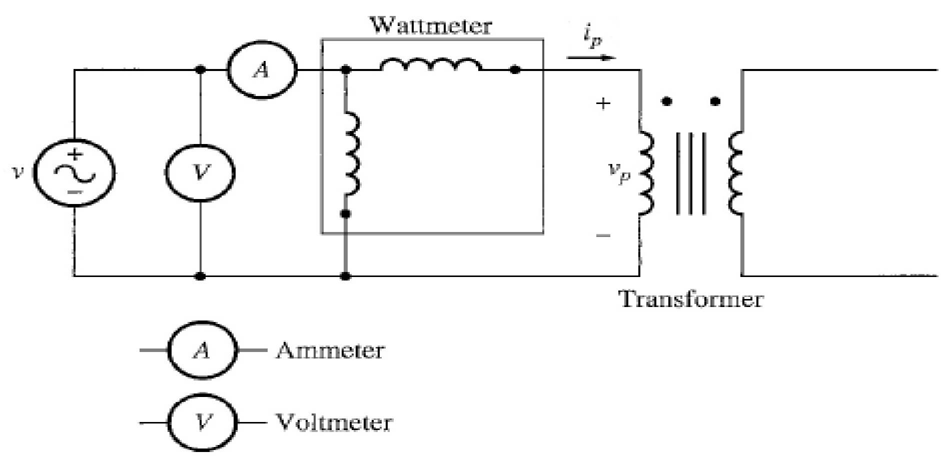
(c) With no excitation branch



(d) With no excitation branch referred to the secondary side

2.4 .1 Determining the Values of Components in the Transformer Model

<u>Open Circuit Test</u>



The transformer's **secondary winding is open-circuited**, and its **primary winding is connected to a full-rated line voltage**.

All the input current will be flowing through the excitation branch of the transformer. The series element R_P and X_P are too small in comparison to R_C and X_M to cause a significant voltage drop. Essentially all input voltage is dropped across the excitation branch. The transformer equivalent circuit becomes:

Full line voltage is applied to the primary – input voltage, input current, input power measured. Then, power factor of the input current and magnitude and angle of the excitation impedance can be calculated.

To obtain the values of R_C and X_M , the easiest way is to find the admittance of the branch.

Conductance of the core loss resistor, $G_C = \frac{1}{R_C}$

Susceptance of the magnetizing inductor, $B_M = \frac{1}{X_M}$

These two elements are in parallel, thus their admittances add.

Total excitation admittance, $Y_E = G_C - jB_M$

The **magnitude** of the excitation admittance (referred to primary),

$$|Y_E| = \frac{I_{OC}}{V_{OC}}$$

The **angle** of the admittance can be found from the circuit power factor as,

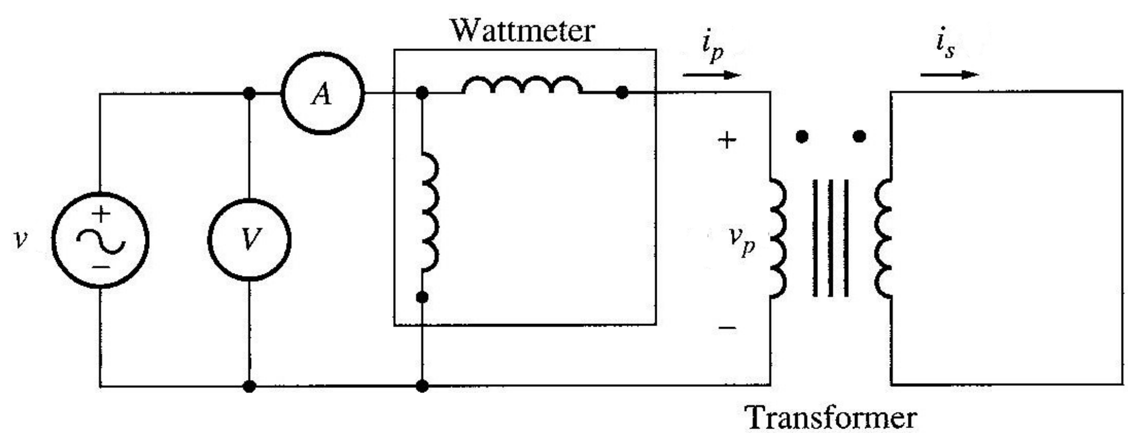
$$\mathrm{PF} = cos\theta = \frac{P_{\mathrm{OC}}}{V_{\mathrm{OC}} I_{OC}}$$

The power factor is always lagging for a real transformer. Therefore,

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle - cos^{-1}\mathrm{PF}$$

This equation can be written in the complex number form and hence the values of R_C and X_M can be determined from the open circuit test data.

Short-Circuit Test



The **secondary terminals are short circuited**, and the **primary terminals** are **connected to a fairly low-voltage source**.

The input voltage is adjusted until the current in the short circuited windings is equal to its rated value. The input voltage, current, and power are measured.

Because voltage is so low, **negligible current flows** through **excitation branch**. Equivalent circuit becomes:

The voltage drop in the transformer can be attributed to the series elements in the circuit.

Magnitude of the series impedances referred to the primary side is

$$|Z_{\mathrm{SE}}| = \frac{V_{\mathrm{SC}}}{I_{\mathrm{SC}}}$$

The current angle can be found from the circuit power factor as,

$$\mathrm{PF} = cos\theta = \frac{P_{\mathrm{SC}}}{V_{\mathrm{SC}} I_{\mathrm{SC}}}$$

The transformer series impedance in phasor form is,

$$Z_{\mathrm{SE}} = \frac{V_{\mathrm{SC}}\angle 0}{I_{\mathrm{SC}}\angle -\theta}$$

The series impedance in rectangular form is,

$$Z_{\mathrm{SE}} = R_{\mathrm{eq}} + jX_{\mathrm{eq}}$$

$$= (R_{\mathrm{P}} + a^2 R_{\mathrm{S}}) + j(X_{\mathrm{P}} + a^2 X_{\mathrm{S}})$$

Note that: The short circuit test only **determines** the **total series impedance referred to** the **primary side**.

There is **no** easy **way to split** the series impedance into their primary and secondary components from these test and the total series impedance is usually adequate.

The same tests can be performed on the secondary side.

Example 2.2

The equivalent circuit impedances of a 20-kVA, 8000/240-V, 60-Hz transformer are to be determined. The open circuit test and the short circuit test were performed on the primary side of the transformer, and the following data were taken:

Open circuit test (on primary)	**Short circuit test**
V_{OC} = 8000 V	V_{SC} = 489 V
I_{OC} = 0.214 A	I_{SC} = 2.5 A
P_{OC} = 400 W	P_{SC} = 240 W

Find the impedance of the approximate equivalent circuit referred to the primary side, and sketch the circuit.

2.5 The Per-Unit System of Measurements

Per-unit system enables:

1. Solving of circuits containing transformers without the need for explicit voltage-level conversions at every transformer

In per-unit system, each **electrical quantity** is **measured as a decimal fraction** of some base level. Any quantity can be expressed on a per-unit basis by the equation

$$\text{Quantity per unit} = \frac{\text{Actual value}}{\text{base value of quantity}},$$

where *actual value* is a value in volts, amperes, ohms, etc.

Customary to **select 2 base values** at a specific point in the system, typically

- Base voltage V_{base}
- Base power P_{base}(or apparent power)

In single phase system, the base quantities can be calculated using electrical laws such as,

$$P_{\text{base}}, Q_{\text{base}}, \text{or } S_{\text{base}} = V_{\text{base}} I_{\text{base}}$$

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}}$$

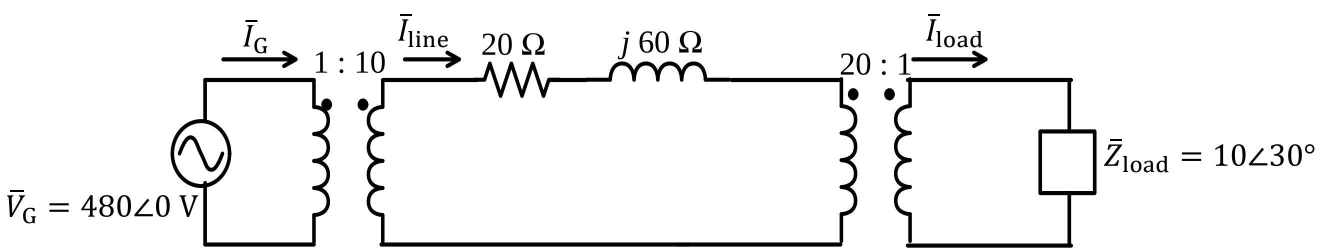
For transformer analysis:

• $\boldsymbol{S}_{\textbf{base}}$ is the **same** at both ends since transformer input power = output power
• $\boldsymbol{V}_{\textbf{base}}$ **changes** at every transformer in the system according to its turns ratio

Hence, P.U. system automatically refers quantities to a common voltage level.

Example 2.3

A 480V generator is connected to an ideal 1:10 step-up transformer, a transmission line, an ideal 20:1 step-down transformer and a load. The impedance of the transmission line is 20+j60Ω and the impedance of the load is $10\angle 30°\Omega$. The base values for this system are chosen to be 480 V and 10 kVA at the generator.



(a) Find the base voltage, current, impedance, and apparent power at every point in the power system.
(b) Convert this system to its per-unit equivalent circuit.
(c) Find the power supplied to the load in this system.
(d) Find the power lost in the transmission line.

If more than one machine and one transformer is used in a single power system, the system based voltage and power can be chosen arbitrarily, but the entire system must have the same base. One common procedure is to choose the system base quantities to be equal to the base of the largest component in the system.

Converting per-unit values from one base to another base can be carried out by converting them to their actual value.

$$(P,Q,S)_{\text{pu on base 2}} = (P,Q,S)_{\text{pu on base 1}} \frac{S_{\text{base1}}}{S_{\text{base2}}}$$

$$V_{\text{pu on base 2}} = V_{\text{pu on base 1}} \frac{V_{\text{base 1}}}{V_{\text{base 2}}}$$

$$(R,X,Z)_{\text{pu on base 2}} = (R,X,Z)_{\text{pu on base 1}} \frac{(V_{\text{base 1}})^2 (S_{\text{base 2}})}{(V_{\text{base 2}})^2 (S_{\text{base 1}})}$$

2.6 Transformer Voltage Regulation and Efficiency

The output voltage of a transformer varies with the load even if the input voltage remains constant because a real transformer has series impedance within it.

In order to compare the transformer output voltage variation to the transformer load, *voltage regulation* VR is defined.

Full-load voltage regulation is defined as,

Since $V_{\mathrm{S}} = \frac{V_{\mathrm{P}}}{a}$, the full-load voltage regulation can be expressed as,

For per-unit system the voltage regulation can be expressed as,

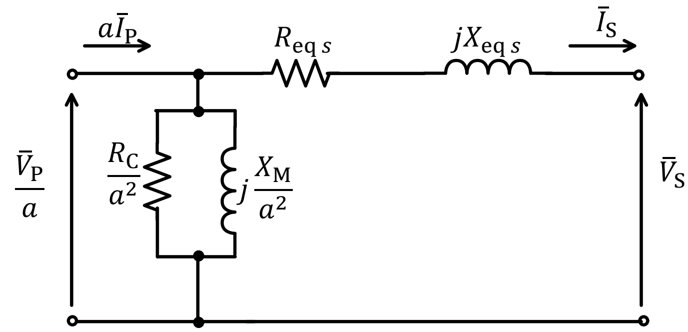
$$\mathrm{VR} = \frac{V_{\mathrm{P,pu}} - V_{\mathrm{S,fl,pu}}}{V_{\mathrm{S,fl,pu}}} \times 100\%.$$

A **low value** of VR is often **desirable**.

2.6.1 The Transformer Phasor Diagram

To determine the voltage regulation of a transformer, it is necessary to consider the voltage drop within it.

Transformer equivalent circuit referred to the secondary side:

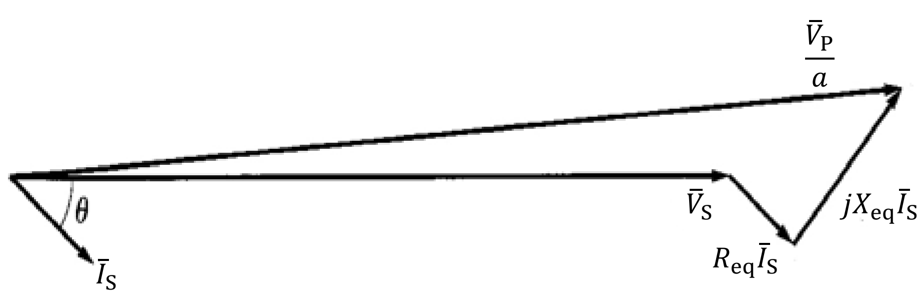


In determining the voltage regulation,

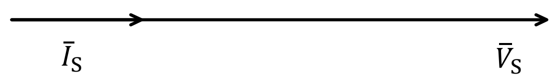
i. The effects of the excitation branch can be ignored
ii. Consider the magnitude of series impedances and
iii. phase angle of the current flowing in the transformer

Phasor diagrams are used to determine the effects of impedances and current phase angles. By applying Kirchhoff's voltage law to the transformer equivalent circuit above, the primary voltage can be found as

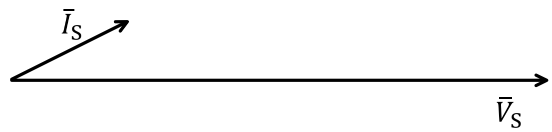
A phasor diagram can be used to represent the equation above. Consider a **lagging power factor**. The phasor voltage $\bar{V}_S$ is assumed to be at zero angle.



At **unity power factor**, VR > 0



At **leading power factor**, VR < 0



2.6.2 Transformer Efficiency

The efficiency of a device is defined as,

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

In **a transformer**, the **losses** present are:

- copper losses,
- hysteresis losses
- eddy current losses

Hence, the efficiency of a transformer at a given load is,

$$\eta = \frac{V_S I_S cos\theta}{P_{\text{Cu}} + P_{core} + V_S I_S cos\theta} \times 100\%$$

Example 2.4

A 15kVA, 2300/230 V transformer is to be tested to determine its excitation branch components, its series impedances, and its voltage regulation. The following data have been taken from the primary side of the transformer:

Open circuit test	*Short-circuit test*
V_{OC}= 2300V	V_{SC}= 47V
I_{OC}= 0.21A	I_{SC}= 6A
P_{OC}= 50W	P_{SC}= 160W

(a) Find the equivalent circuit referred to the high voltage side
(b) Find the equivalent circuit referred to the low voltage side
(c) Calculate the full-load voltage regulation at 0.8 lagging PF, 1.0 PF, and at 0.8 leading PF.
(d) Find the efficiency at full load with PF 0.8 lagging.

2.7 Review of three-phase circuits

Three-phase power system consists of **3 generators.** Each generator is connected to a **load** through a **transmission line**.

Why three-phase?

1. get more power per kilogram of metal from a machine
2. constant power is delivered at all times

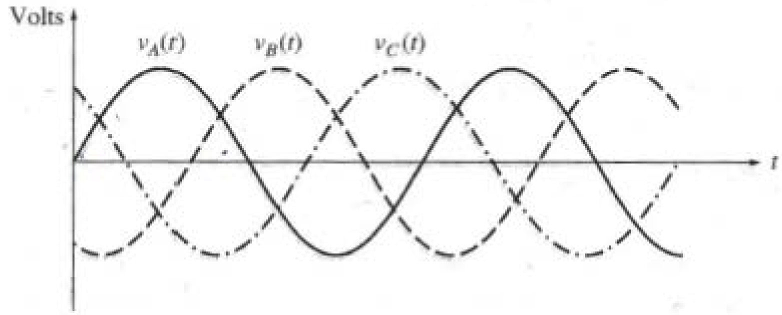
The generators supply **voltages** that are **equal** in **magnitude** but **differing** in **phase angle** by **120**°, i.e.

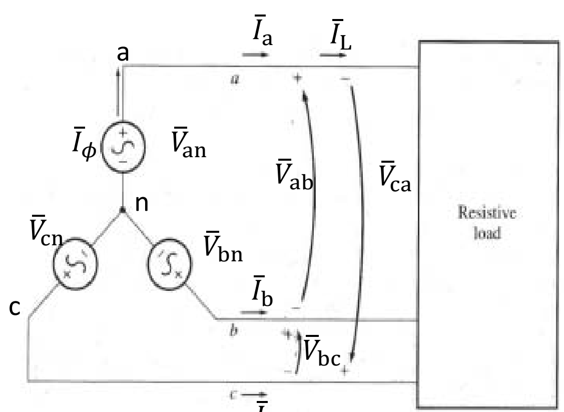
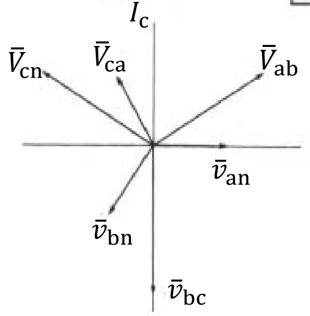
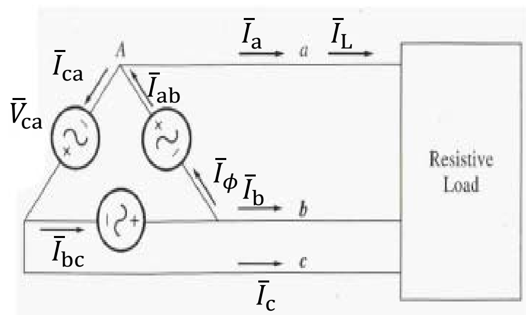
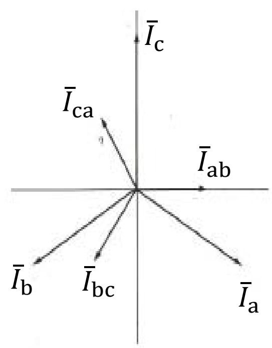
$\bar{V}_{\mathrm{A}} = V\angle 0°$ $\quad v_{\mathrm{A}} = \sqrt{2}V sin\omega t$ V

$\bar{V}_{\mathrm{B}} = V\angle -120°$ $\quad v_{\mathrm{B}} = \sqrt{2}V sin(\omega t - 120°)$ V

$\bar{V}_{\mathrm{C}} = V\angle -240°$ $\quad v_{\mathrm{C}} = \sqrt{2}V sin(\omega t - 240°)$ V

The voltage waveform for the three phases:



Wye (Y) Connection	**Delta (Δ) Connection**
 	 
$\bar{V}_{ab} = \bar{V}_a - \bar{V}_b$ $= V_\phi\angle 0^\circ - V_\phi\angle -120^\circ$ $= V_\phi - \left(-\frac{1}{2}V_\phi - j\frac{\sqrt{3}}{2}V_\phi\right)$ $= \sqrt{3}V_\phi\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)$ $\bar{V}_{ab} = \sqrt{3}V_\phi\angle 30^\circ$ (for *abc* phase sequence)	$\bar{I}_a = \bar{I}_{ab} - \bar{I}_{ca}$ $= I_\phi\angle 0^\circ - I_\phi\angle -240^\circ$ $= I_\phi - \left(-\frac{1}{2}I_\phi + j\frac{\sqrt{3}}{2}I_\phi\right)$ $= \sqrt{3}I_\phi\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)$ $\bar{I}_a = \sqrt{3}I_\phi\angle -30^\circ$ (for *abc* phase sequence)

A simple concept that all students must remember is that:

Wye (Y) Connection	**Delta (Δ) Connection**
$V_{\phi} = \dfrac{V_{\mathrm{L}}}{\sqrt{3}}$	$V_{\phi} = V_{\mathrm{L}}$
$I_{\phi} = I_{\mathrm{L}}$	$I_{\phi} = \dfrac{I_{\mathrm{L}}}{\sqrt{3}}$
$S_{1\phi} = \dfrac{S_{3\phi}}{3}$	$S_{1\phi} = \dfrac{S_{3\phi}}{3}$

2.8 Three Phase Transformers

The primaries and secondaries of any three-phase transformer can be independently connected in either a wye (Y) or a delta (Δ).

The key to analyzing any 3-phase transformer bank is to look at a single transformer in the bank. The impedance, voltage regulation, efficiency, and similar calculations for a 3-phase transformer are done on a per-phase basis, using the same technique learnt previously.

For example a Y-Δ transformer connection, the voltage ratio of each phase is,

$$\frac{\bar{V}_{\phi\mathrm{P}}}{\bar{V}_{\phi\mathrm{S}}} = a.$$

The overall relationship between line voltage on the primary side and the line voltage on the secondary side is,

$$\frac{\bar{V}_{\mathrm{LP}}}{\bar{V}_{\mathrm{LS}}} = \frac{\sqrt{3}\bar{V}_{\phi\mathrm{P}}}{\bar{V}_{\phi\mathrm{S}}} = \sqrt{3}a$$

The same method can be carried out for other transformer connections.

The Per-unit System for 3-Phase Transformer

The per unit system of measurements application for 3-phase is the same as in single phase transformers. The single-phase base equations apply to 3-phase on a per-phase basis.

Say the total base voltampere value of a transformer bank is called S_{base}, then the base voltampere value of one of the transformer is

$$S_{1\phi,base} = \frac{S_{base}}{3}$$

And the base current and impedance are

$$I_{\phi,base} = \frac{S_{1\phi,base}}{V_{\phi base}} \qquad Z_{base} = \frac{\left(V_{\phi,base}\right)^2}{S_{1\phi,base}}$$

$$I_{\phi,base} = \frac{S_{base}}{3V_{\phi base}} \qquad Z_{base} = \frac{3\left(V_{\phi,base}\right)^2}{S_{base}}$$

<u>Example 2.9</u>

A 50-kVA 13,800/208-V Δ-Y distribution transformer has a resistance of 1% and a reactance of 7% per unit.

a. What is the transformer's phase impedance referred to the high voltage side?
b. Calculate this transformer's voltage regulation at full load and 0.8PF lagging, using the calculated high side impedance.
c. Calculate this transformer's voltage regulation under the same conditions, using the per-unit system.